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## Motivations

The major difficulty that are facing transport models is the formation of clusters. For this reason, this aspect is often oversimplified, when not omitted.

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- Having the clusters correctly formed is as important as the transport and creation of their constituents in the curse of the collisions.

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- Having the clusters correctly formed is as important as the transport and creation of their constituents in the curse of the collisions.
- Because, apart from emitted elementary particles, they carry the only information that the experimental instruments can measure.

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- Having the clusters correctly formed is as important as the transport and creation of their constituents in the curse of the collisions.
- Because, apart from emitted elementary particles, they carry the only information that the experimental instruments can measure.
- Making clusters is not an easy task, because it involves, in a complex environment:
  - the fundamental nuclear properties,
  - quantum effects,
  - and variable timescales.



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a) Take the positions and momenta of all nucleons at time t.

b) Combine them in all possible ways into fragments or leave them as single nucleons.

c) Neglect the interaction among clusters.

d) Choose that configuration which has the highest binding energy. Simulations show: Clusters chosen that way at early times are the prefragments of the final state clusters, because fragments are not a random collection of nucleons at the end but initial-final state correlations.



Simulated Annealing Procedure: PLB301:328,1993; later called SACA.

2 steps:

1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.



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2) Take randomly 1 nucleon out of one fragment  $E = E^{1}_{kin} + E^{2}_{kin} + V^{1} + V^{2}$ 3) Add it randomly to another fragment  $E' = E^{1'}_{kin} + E^{2'}_{kin} + V^{1} + V^{2'}$ 

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<u>If E' < E</u> take the new configuration <u>If E' > E</u> take the old with a probability depending on E'-E Repeat this procedure very many times... It leads automatically to the most bound configuration.



\* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390

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➡ With MST, one has to consider necessarily later times (typically 200-400 fm/c), where the dynamical conditions are no longer the same.

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Unlike SACA, MST is not able to describe the early formation of fragments.

 With MST, one has to consider necessarily later times (typically 200-400 fm/c), where the dynamical conditions are no longer the same.
Advantage of SACA : the fragment partitions can reflect the early dynamical conditions (Coulomb, density, flow details, strangeness...).

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#### Toward the isotope yields... IQMD + new SACA

SACA is applied here on the IQMD transport model\* calculations \*C. Hartnack *et al.*, Eur. Phys. J. A1, 151(1998).

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#### Au+Au at 100 A.MeV - b=7 fm





Large Z range of measured isotopes, large flow, high excitation energy.



#### An example of complex system accurately measured

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#### An example of complex system accurately measured



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- At this stage, SACA contains as ingredients of the potential making the binding energy of the clusters :
- volume component: mean field (Skyrme, dominant)
  correction of surface effects: Yukawa



SACA with asymmetry energy

IQMD <sup>136</sup>Xe+<sup>112</sup>Sn at 100 A.MeV, b=1 fm, t <sub>SACA</sub>=60 fm/c



А

18

A

10 12 14 16

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10

12 14

А

8

10-

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0

2

4 6

10

2

4 6 8

0
SACA with asymmetry energy

IQMD <sup>136</sup>Xe+<sup>112</sup>Sn at 100 A.MeV, b=1 fm, t <sub>SACA</sub>=60 fm/c



Here,

in IQMD and SACA, we adopt the following asymmetry energy parametrisation:

 $E_{asy} = E_0.(<\rho_B > /\rho_0)^{(\gamma-1)}.(<\rho_n > - <\rho_p >) / <\rho_B >$ with E<sub>0</sub>=32 MeV,  $\gamma$ =1 («stiff»)

 $\Rightarrow$  Z and A yields not strongly modified

➡ Isotope yields shrink onto the N=Z line

Still not fully realistic: shell, odd-even effects (pairing) still absent.



#### SACA with asymmetry energy and pairing



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## **Excitation energy of the primary fragments** $E^* = E_{g.s.} - E_{bind}$



#### **Excitation energy of the primary fragments**





IQMD+SACA  $^{129}$ Xe+ $^{124}$ Sn at 100 A.MeV - b < 0.2 b  $_{0}$  fm (central)

On Average, 2 A.MeV of excitation energy. Corresponds to findings of S. Hudan et al. (INDRA collaboration), PRC 67, 064613 (2003). => secondary decay (GEMINI) justified here.

### **Excitation energy of the primary fragments**



At relativistic energies, in the participant-spectator regime, heavy primary clusters are produced colder on average.

# What can we learn from the isotope yields regarding the asymmetry energy?

Mean radius of primary clusters

IQMD+SACA

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Though the medium is dense at this early stage, the dense clusters are disfavoured, because they would correspond to nucleons flowing against each other, hence with too high relative momenta to make a cluster.

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W. Reisdorf and the FOPI Collaboration

Nuclear Physics A 848 (2010) 366–427





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=> At high energy, the asymmetry energy effect on clusters seems to vanish. Timescale effect? Non-linear dependence on the density?

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Extending SACA for clusterising hadrons with hyperons (lambdas,...) for making hypernuclei is straightforward:

- \* one replaces  $V_{n-p}$  by  $V_{\Lambda-p}$  and  $V_{n-n}$  by  $V_{\Lambda-n}$
- \* and applies with these modifications the SACA algorithm.

If in a final fragment there is a lambda, a hypernucleus should be created. As a first approach, we have adopted  $V_{\Lambda-N} = 2/3 V_{n-N}$ ; further refinements are possible.

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Many ways of producing lambdas:  $K N \to \Lambda \pi, \pi^+ n \to \Lambda K^+, \pi^- p \to \Lambda K_0, p p \to \Lambda X$  $\Rightarrow$  influence of the EOS, in medium-properties, etc.













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Preliminary





## Strong phase space constraints IQMD+SACA <sup>58</sup>Ni+<sup>58</sup>Ni at 101 A C M

1.91 A.GeV (b < 6 fm) t<sub>cluster.</sub>=20 fm/c

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Soft EOS, no m.d.i., with Kaon pot.





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IQMD+SACA  ${}^{58}Ni+{}^{58}Ni$  at 1.91 A.GeV (b < 6 fm) - t<sub>cluster.</sub>=20 fm/c

 $\Lambda t$ 

p

n

Λ

IQMD+SACA <sup>58</sup>Ni+<sup>58</sup>Ni at 1.93 A.GeV (b < 6 fm,  $t_{cluster.}$  = 20fm/c) - soft no mdi, kaon pot.



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## **Summary and perspectives**

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Summary:

Supplying SACA with a more precise description of nuclei binding energy at abnormal density allows promising, realistic predictions of absolute isotope yields, and hypernuclei.
The asymmetry energy has a strong influence on the anisotropy (apparent stopping power) for some isotopes (<sup>3</sup>He, <sup>4</sup>He, ...).

\* Within this model, isotope yields cannot inform on the high density dependence of the asymmetry energy.  $\Rightarrow$  better look at n / p, K<sup>+</sup> / K<sup>-</sup>,  $\pi^+$  /  $\pi^-$  yields/flows for that purpose.

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#### Further developments:

After processing SACA, proceed:

✤ further decay of primary unstable isotopes like <sup>8</sup>Be, <sup>5</sup>He, etc., which lifetime do not allow to detect them still bound in the detectors,

\* allow early  ${}^{3}\text{He} + n \rightarrow {}^{4}\text{He}$  according to its particularly high cross-section.

★ secondary decay (evaporation code like GEMINI) of still excited clusters. Particularly relevant at intermediate energies (E<sub>beam</sub> 100 A.MeV down to the Fermi regime)

\* for hypernuclei formation, refine lambda-N potential in SACA or EOS/Kaon potential in IQMD in order to predict reasonnably the measured cross-sections, and momentum distributions, which are very constraining.

Do the pairing and shell effect affect the primary fragments?

Probably yes, because:

 $\checkmark$  according to E. Khan et al., NPA 789 (2007) 94, pairing vanishes above T  $\approx 0.5 \Delta_{\text{pairing}}$ 

 $\Delta_{\text{pairing}}(\rho_0) = 12 \text{ MeV}/\sqrt{A}; \Rightarrow \Delta B_{\text{pairing}}(^4\text{He}) = 12 \text{ MeV}, \Delta B_{\text{pairing}}(^3\text{He}) = 6.9 \text{ MeV}.$ 

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→Need to apply a pairing correction factor depending on the cluster density.

E. Khan, M. Grasso and J. Margueron, in PRC 80 (2009) 044328 have derived the following function

from the pairing potential  $V_{pair} = V_0 \{1 - \eta [\rho(r)/\rho_0]\} \delta(r_1 - r_2)$  within the Hartree-Fock-Bogolioubov method:



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#### More detailed structure corrections to apply

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→  $\Delta B_{pairing}(N,Z,\rho_0)$ . And for a cluster at mean baryonic density  $\langle \rho_B \rangle$ , it will be



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80

100

120 140

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 $\Delta B_{\text{pairing}}(N,Z,\rho_0) \ge f_{\eta}(<\rho_B>)$ 

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N120

100

20

MMMC mass table - binding energy per nucleon (MeV) vs (N,Z

100

120

#### SACA with asymmetry energy and pairing



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#### SACA with asymmetry energy and pairing



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Illustration: alpha particles in



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J. Lukasik ALaDiN-INDRA Coll.

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