ON THE SEQUENCE OF PROTON AND COMPOSITE PARTICLE EMISSION IN CENTRAL COLLISIONS OF Ru(Zr)+Ru(Zr) AT 400 $A{\cdot}$MeV

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Abstract

Small-angle correlations of pairs of nonidentical light charged particles produced in central collisions of Ru and Zr at a beam energy of 400 $A{\cdot}$MeV are investigated with the FOPI detector system installed at the heavy-ion synchrotron SIS at Darmstadt. The difference of longitudinal correlation functions with the relative velocity parallel and anti-parallel to the center-of-mass velocity of the pair in the source frame allows extracting the apparent emission-time sequence of the charged particles. The comparison of the correlations with results of a final-state interaction model delivers quantitative estimates of these emission times. The strong collective expansion of the participant zone introduces not only an apparent reduction of the source radius but also a modification of the emission time differences. After correcting for both effects a complete sequence of emission times of p, d, t, ^{3}He and α particles is presented here for the first time.

1 Introduction

Correlations of two nonidentical particles at small relative velocities are - due to final state interactions - sensitive to the space-time structure of the emission points. Usually, the correlation is used to extract the size of the source and the time duration of the emission [1, 2, 3, 4]. However, it contains also information on the emission time differences of the two particles. Lednický et al. [5] have proposed to use the above sensitivity of the correlation function to measure directly the space-time differences in the emission of particles of different types. This is possible, since the wave functions for nonidentical particles are asymmetric with respect to the forward and backward direction of the lighter particle. Assume the particles of the pair have a small relative velocity. If the faster particle approaches and passes the other one from behind the pair would experience a stronger final-state interaction than in case that it had already started in front of the slower one. Thus, the ratio of the correlation function in forward and backward direction is related to the formation sequence.

First theoretical [6] and experimental work [7] concentrated on the forward-backward asymmetry of produced proton-pion pairs. In the present work [8] the proposed method is applied to measure the pair-wise space-time differences of light charged particles produced in symmetric central collisions of $A \simeq 100$ nuclei at a projectile energy of 400 MeV per nucleon.

2 The experiment

The experiment has been performed at the heavy-ion synchrotron SIS at GSI Darmstadt. Targets of 1 % interaction thickness of ^{96}Ru and ^{96}Zr have been irradiated by ^{96}Ru and ^{96}Zr ions of $400A\cdot$MeV beam energy. The present analysis uses a subsample of the data, taken with the outer Plastic Wall/Helitron combination of the FOPI detector system [9]. The Plastic Wall and the Helitron have full overlap for polar angles between 8.5 degrees and 26.5 degrees. The Plastic Wall delivers - via energy loss vs. time-of-flight (TOF) measurement - the nuclear charge Z and the velocity β of the particles. The Helitron gives the curvature (which is a measure of the momentum over charge (p/Z)) of the particle track in the field of a large superconducting solenoid. The mass m is determined via $mc = (p/Z)_{Hel}/(\beta\gamma/Z)_{PlaWa}$, where $\gamma = (1-\beta^2)^{-1/2}$.

About $4 \cdot 10^6$ central collisions are selected by demanding large charged-particle multiplicities to be measured in the outer Plastic Wall. The corresponding integrated cross section comprises about 8 % of the total cross section. For this centrality class one would expect - within a geometrical picture - an average impact parameter of about 2 fm.

3 Correlation function

Let $Y_{12}(\mathbf{p}_1, \mathbf{p}_2)$ be the coincidence yield of pairs of particles having momenta $\mathbf{p}_1$ and $\mathbf{p}_2$. Then the two-particle correlation function is defined as

$$1 + \mathrm{R}(\mathbf{p}_1, \mathbf{p}_2) = \mathcal{N} \frac{\sum_{events,pairs} Y_{12}(\mathbf{p}_1, \mathbf{p}_2)}{\sum_{events,pairs} Y_{12,mix}(\mathbf{p}_1, \mathbf{p}_2)}. \tag{1}$$

The sum runs over all events fulfilling the above mentioned global selection criterion and over all pairs satisfying certain conditions given below. Event mixing, denoted by the subscript "mix", means to take particle 1 and particle

2 from different events. $\mathcal{N}$ is a normalization factor fixed by the requirement to have the same number of true and mixed pairs. The correlation function (1) is then projected onto the relative momentum $\mathbf{q} = \mu \mathbf{v}_{12} = \mu(\mathbf{v}_1^{cm} - \mathbf{v}_2^{cm})$. Here, $\mathbf{v}_i^{cm}$ with their projections v_L on the pair velocity $\mathbf{V}$ are the particle velocities in the c.m. system of the colliding nuclei and μ is the reduced mass of the pair. Introducing the angle ζ between $\mathbf{q}$ and the c.m. sum momentum of the particle pair $\mathbf{P}_{12}^{cm} = \mathbf{p}_1^{cm} + \mathbf{p}_2^{cm}$ the forward and backward correlation functions R^+ and R^- are defined by cuts on the angle ζ, $\cos\zeta > 0$ and $\cos\zeta < 0$, respectively. Then we define the asymmetry by the ratio

$$\mathrm{R}^+(q)/\mathrm{R}^-(q) = \frac{1 + \mathrm{R}(q, v_{L,1} > v_{L,2})}{1 + \mathrm{R}(q, v_{L,1} < v_{L,2})} \tag{2}$$

as a function of the relative momentum.

4 Correlations from final-state interaction

Here, we present a model description similar to that used by Boal et al. [4]. The correlation function of two particles 1 and 2 which move with a pair velocity $\mathbf{V}$ is defined [2, 10] as

$$1 + R(\mathbf{p_1}, \mathbf{p_2}) = \int dt_1 dt_2 d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1, t_1)\rho_2(\mathbf{r}_2, t_2)|\Psi_\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{V}(t_1 - t_2))|^2 \tag{3}$$

where the density $\rho_{1,2}(\mathbf{r}, t)$ describes the probability for the emission of particle 1 or 2, respectively, at time t and spatial coordinate $\mathbf{r}$. The wave function $\Psi_\mathbf{q}$ describes the relative motion of the two particles which depends only on the relative coordinate $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ and on the relative momentum $\mathbf{q}$.

First, we assume that the source functions $\rho_{1,2}$ do not depend on the velocity and have a Gaussian shape in space and time characterized by a radius parameter R_0 and emission times $\tau_{1,2}$ while the duration of the emission is given by τ:

$$\rho_{1,2}(\mathbf{r}, t) = \frac{1}{4\pi^2 R_0^3 \tau} \exp[-\frac{r^2}{2R_0^2} - \frac{(t - \tau_{1,2})^2}{2\tau^2}]. \tag{4}$$

Integrating over center-of-mass coordinate and time variables we obtain

$$1 + R(\mathbf{p_1}, \mathbf{p_2}) = \int d^3r S(\mathbf{r}, \mathbf{V})|\Psi_\mathbf{q}(\mathbf{r})|^2 \tag{5}$$

with

$$S(\mathbf{r}, \mathbf{V}) = \frac{1}{(4\pi)^{3/2} R_0^2 D} \exp\Big(- \frac{1}{4D^2}\big[(\mathbf{V}\Delta\tau + \mathbf{r})^2 + \frac{\tau^2}{R_0^2}(r^2 V^2 - (\mathbf{r}\cdot\mathbf{V})^2)\big]\Big), \tag{6}$$

where we have introduced the emission time difference $\Delta\tau$ and an effective radius D by

$$\Delta\tau = \tau_1 - \tau_2, \quad D = \sqrt{R_0^2 + (V\tau)^2}. \tag{7}$$

The correlation depends on the sign of emission time difference $\Delta\tau$ if the two particle wave function $\Psi_{\mathbf{q}}$ contains terms of an odd power of the scalar product $\mathbf{q}\cdot\mathbf{r}$ which is possible for nonidentical particles.

The wave functions $\Psi_{\mathbf{q}}$ in Eq. (3) are generated by partial wave expansion technique as solutions of the corresponding radial Schrödinger equations with properly chosen parameters of Woods-Saxon potentials, similarly as in ref. [4].

In a second step we consider the effect of the collective radial flow. There is good experience [11] with the "nuclear Hubble scenario" which suggests a linearly increasing velocity profile. Using again a Gaussian density profile we write for the emission function (4)

$$\rho_{1,2}(\mathbf{v},\mathbf{r},t) \sim \exp[-\frac{r^2}{2R_{1,2}^2} - \frac{(t-\tau_{1,2})^2}{2\tau^2} - \frac{A_{1,2}m_0(\mathbf{v}-\eta\mathbf{r})^2}{2T}], \tag{8}$$

where the quantities m_0, $R_{1,2}$ represent the nucleon mass and the source radii for the different particles. The velocities of the particles are thermally distributed, characterized by the temperature T, around a radial flow velocity defined by the "nuclear Hubble constant" η which is related to the flow energy via $\epsilon_{flow} = 3m_0\eta^2R_0^2/2$.

Repeating the integration of Eq. (3) with the new functions $\rho_{1,2}$ one arrives at a similar relation for the function S as Eq. (6), however the parameters R_0, $\Delta\tau$ and D of Eq. (7) have to be replaced by quantities indicated by an asterisk:

$$\Delta\tau^* = \Delta\tau + \Delta\tau^{flow} \tag{9}$$

with

$$\Delta\tau^{flow} = -\Big[\frac{\tilde{r}_1^2A_1}{1+\epsilon\tilde{r}_1^2A_1} - \frac{\tilde{r}_2^2A_2}{1+\epsilon\tilde{r}_2^2A_2}\Big]\sqrt{\epsilon\frac{m_0}{T}R_0^2} \tag{10}$$

and

$$R_0^* = \sqrt{\frac{1}{2}(\frac{R_1^2}{1+\epsilon\tilde{r}_1^2A_1} + \frac{R_2^2}{1+\epsilon\tilde{r}_2^2A_2})}, \quad D^* = \sqrt{(R_0^*)^2 + (V\tau)^2}. \tag{11}$$

Here, we use the abbreviations $\tilde{r}_{1,2} = R_{1,2}/R_0$, and $\epsilon = \epsilon_{flow}/\epsilon_{therm}$ is the ratio of the radial flow energy ϵ_{flow} and the thermal energy $\epsilon_{therm} = \frac{3}{2}T$.

The essential result is that even if the two particles are emitted from the same source region, $R_1 = R_2$, at the same time, $\tau_1 = \tau_2$, the flow causes still an effective time difference $\Delta\tau^* = \Delta\tau^{flow}$ if the two particle masses are different.

5 Results and comparison with model

5.1 The source extension

In fig. 1 (left panel) the angle-integrated two-proton correlation function is compared to the model prediction for an emission from a source of zero lifetime and Gaussian radius $R_0 = R_{rms}/\sqrt{3}$. The theoretical correlation function is folded with an experimental resolution function of Gaussian shape with the dispersion $\delta(q) = \mu \cdot 0.006$ c. The apparent reduction (11) of the source radius due to radial expansion effects is estimated to $R_0^*/R_0 = 1/\sqrt{1+\epsilon} = 1/\sqrt{2}$. The best agreement of experimental data and model calculations is found for $R_0 = 5.7$ fm. Though the sensitivity to radius variations of the two-deuteron correlation function (right panel of Fig. 1) is less high, a very similar radius is deduced. Also the correlation functions of nonidentical particles with larger reduced masses can be well reproduced with the same source radius.

For the determination of the emission time differences in the subsequent section the source radii are kept fixed to values of $R_1 = R_2 = R_0 = 5.7$ fm.

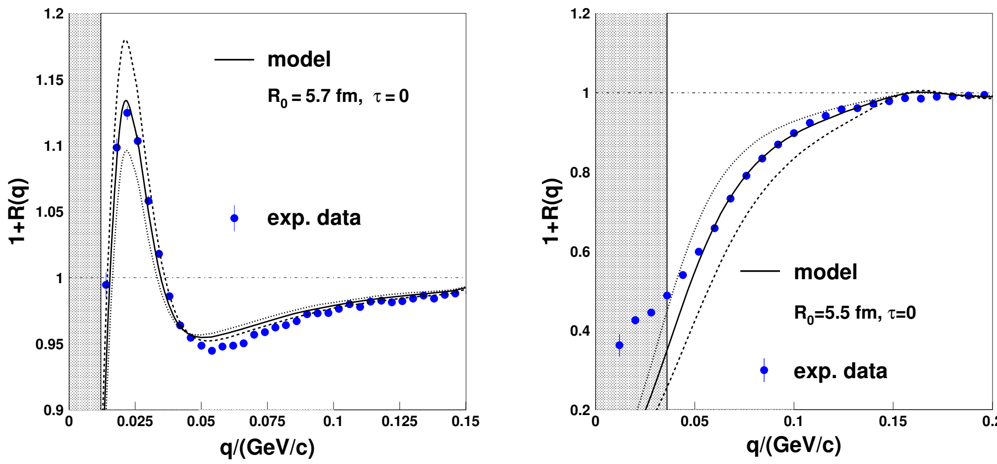

Figure 1: Left panel: The experimental correlation function of proton pairs from central trigger events (dots). The hatched area indicates the unreliable region which may be contaminated by doubly counted scattered particles. The full line represents the model prediction for an emission from a Gaussian source of zero lifetime and radius $R_0 = 5.7$ fm. The dashed and dotted lines are the corresponding results for source radii which differ from the optimum one by -0.3 fm and +0.3 fm, respectively. Right panel: The two-deuteron correlation function compared to the model prediction for zero lifetime and Gaussian radius of $R_0 = 5.5$ fm (full line). The dashed and dotted lines are the corresponding results for source radii which differ from the optimum one by -1 fm and +1 fm, respectively.

The energy ratio and the temperature are chosen to be $\epsilon = 1$ and $T = 37$ MeV, respectively, compatible with the values found in Au on Au collisions [11]. For simplification the duration of the emission τ is set to zero. A finite value leads only to a rescaling of the effective radius in Eq. (11).

5.2 Emission time differences

Out of the ten different combinations of pairs of nonidentical particles which can be constructed from p, d, t, ^{3}He, and α particles there exist only two correlation functions (d-p, ^{3}He-t) which do not contain contributions of resonance decay products. All other correlation functions contain resonance contributions of particle-unbound ground states or excited states of heavier clusters decaying into pairs of light charged particles. In case of the appearance of strong and narrow resonances in the q region of interest, the corresponding theoretical correlation functions are corrected for the experimental q resolution similarly as for p-p correlations.

As an example for resonance-free correlation functions the left part of Fig. 2 gives the ^{3}He-t correlations. Obviously, the suppression at low relative mo-

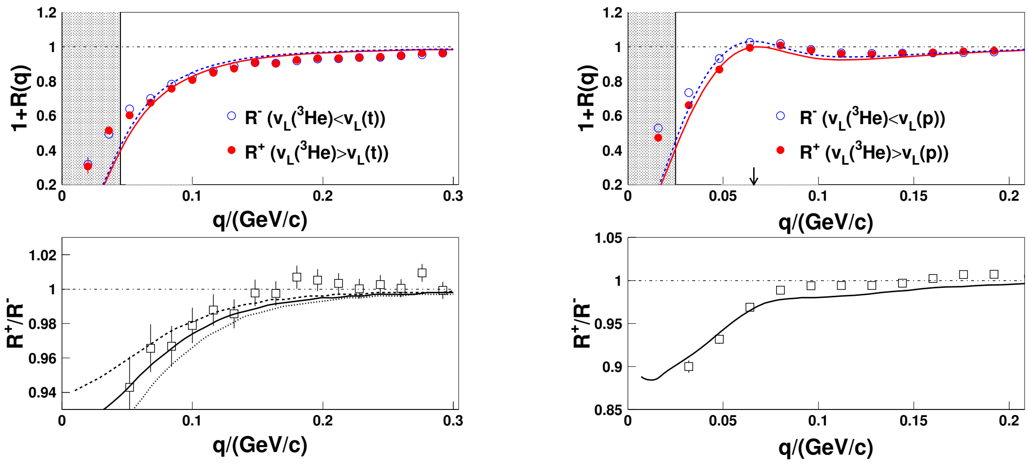

Figure 2: Upper left panel: Forward (full dots) and backward (open dots) longitudinal experimental correlation functions of ^{3}He-t pairs. The full and dashed lines give the corresponding model predictions with the time delay of table 1. Lower left panel: Ratio of forward/backward experimental correlation functions (open squares). The full line represents the ratio of the simulated correlation functions. The dashed and dotted lines give the model predictions for times differences deviating by -0.5 fm/c and +0.5 fm/c from the optimum one, respectively. Right panels: The same for ^{3}He-p pairs. The position of the relevant resonance is marked by an arrow.

Table 1: The apparent and the true emission time differences as determined for all combinations of nonidentical charged-particle correlation functions.

light-charged particle combination 1 - 2	apparent time delay $\Delta\tau^*$ (fm/c)	true time delay $\Delta\tau = \tau_1 - \tau_2$ (fm/c)
d - p	1.7 ± 1	6.5 ± 1
t - p	-4 ± 3	3 ± 3
^{3}He - p	4.5 ± 1	11.7 ± 1
α - p	-3 ± 3	6 ± 3
t - d	-1 ± 1	1.4 ± 1
^{3}He - d	3 ± 1	5.4 ± 1
α - d	-2 ± 1	1.8 ± 1
^{3}He - t	2 ± 1	2 ± 1
α - t	-3 ± 1	-1.5 ± 1
^{3}He - α	4.5 ± 1	3 ± 1

menta gets stronger for ^{3}He particles which are faster than tritons. Since the masses of the two species are practically identical, this correlation function is the only one where the flow correction (10) vanishes and, consequently, the true and the apparent time delays are identical. Obviously, the ^{3}He particles are emitted slightly later than the tritons (about 2 fm/c, cf. table 1).

If a resonance contribution is dominating in the two-particle yield the ratio R^+/R^- is necessarily forced to unity for relative momenta q approaching the resonance value q_i. This is due to the fact that for pure two-body decays both particles are emitted at the same time (and position). The right part of Fig. 2 represents - as an example for resonance containing correlation functions - the results for ^{3}He-p correlations. One broad maximum shows up which is related to the particle-unbound ground state of ^{4}Li ($J^\pi = 2^-$, $\Gamma = 6$ MeV, $q_0 = 66.0$ MeV/c). Here, about 60 % of the true time difference of about 12 fm/c (cf. table 1) arise from the radial flow correction (10).

6 Discussion and Summary

The optimum sequence is derived from a weighted least-squares solution [8] of the ten linear equations for the pair-wise time delays given in table 1. (The precision of the method is about 0.8 fm/c.) For a fixed source radius, the most probable time order of the emission is found as follows: Protons are emitted first, whereas the d, α, t, and ^{3}He particles follow - on average - after a time of 6.3, 7.7, 8.5, and 11.1 fm/c, respectively.

As a consequence of the duality of space and time coordinates, the time delays can be transformed into position differences. When assuming a common emission time for all particles, the time sequence translates into the emission from different source radii R_i according to Eq. (10) as a consequence of the radial flow. Then, the protons, which are emitted earliest in the above time-ordered picture, would come from the most extended source ($R_{rms} = \sqrt{3}R_0 \simeq$ 10 fm) whereas the clusters (with the exception of α particles, which obviously play a special role) are emitted from source radii which decrease with increasing mass. Thus, the radii of the deuterons, α's, tritons, and ^{3}He particles would be about 70%, 64%, 60%, and 47% of that of the protons, respectively.

In conclusion, we have presented experimental correlation functions of non-identical light charged particles produced in central collisions of Ru and Zr at 400 A·MeV. For the first time a sequence of emission times of p, d, α, t and ^{3}He particles has been set up by comparing correlations of particles with relative momenta parallel and anti-parallel to the center-of-mass velocity of the pair in the source system. Collective radial expansion of the participant zone leads to an apparent reduction of the source radius and additional shifts of the emission times. Correcting for both effects we found typical time delays of a few fm/c.

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